this leads to a very low limit of abundance of elements heavier than the iron group. This is also in agreement with satellite results of Kurnosova *et al.*[27]

Two hundred and ninety of the heavy nuclei had energies <700 MeV/N. In addition, we found 227 stopping α particles. Any antiparticle among this group would have certainly been identified. None was found.

[27] L. V. Kurnosova, L. A. Rozorenov, and M. I. Fradkin. Iskusst. Sputniki Zemli 2, 70 (1958).

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Empirical Relations Involving the Hyperons and Baryon Isobars*

R. M. STERNHEIMER
Brookhaven National Laboratory, Upton, New York
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A scheme for the classification of the nucleon isobars recently proposed by Kycia and Riley is extended to include the hyperon isobars $Y_0{}^*$ and $Y_1{}^*$ with strangeness $S=-1$. The present model is essentially based on the existence of empirical mass relations involving the various baryon isobar states. The extent of the validity of these mass relations is discussed. A classification of the six isobar systems which occur in the present scheme is described. We also discuss the set of quantum numbers which is necessary to characterize an isobaric state.

I. INTRODUCTION

In a recent paper, Kycia and Riley[1] have discussed several interesting mass relations which enabled them to classify the nucleon and the known nucleon isobars into two systems, with the nucleon and the $I=J=\frac{3}{2}$ isobar as the ground states of the two systems. Subsequently, Sternheimer[2] pointed out some additional mass relations involving the hyperons and baryon isobars.

In Sec. II of the present paper, we will discuss a scheme similar to that of Kycia and Riley, which enables one to classify the $Y_0{}^*$ and $Y_1{}^*$ isobars of strangeness $S=-1$. The present model is essentially based on the existence of an empirical mass sum relation, according to which the mass of an isobar is equal, to a good accuracy, to the sum of the masses of a baryon ground state and one or several mesonic particles. In Sec. III, the extent of the validity of the mass relation is investigated.

In Sec. IV, it will be shown that the mass relation is also valid for several mesonic particles (including the η meson), which can be regarded as combinations of a small number of pions. In Sec. V, we will discuss a possible interpretation of these empirical results, as indicating that the baryon isobars can be considered as loosely bound nuclei consisting of an unexcited baryon plus one or several mesonic particles. A similar interpretation can be given for the π meson combinations which are used as constituents of some of the isobars in the present scheme.

In Sec. VI, a classification of the isobar systems is proposed. It is also pointed out that the number of two-particle thresholds which do not lead to a maximum in the total πN or $\bar{K}N$ cross section greatly exceeds the number of thresholds which can be correlated with such a maximum (isobar formation). This fact may indicate the existence of a selection rule which determines under what conditions a given baryon state and mesonic particle can combine to form an isobar. In Sec. VII, we examine the set of quantum numbers which is necessary in order to characterize an isobaric state. It is found that the set I, J, L, S (I=isotopic spin, J =total angular momentum, L=orbital angular momentum, S=strangeness) will, in general, be necessary and sufficient to define an isobaric state, i.e., there will be, at most, one isobar with given values of I, J, L, and S. This result indicates that there is probably no analog to a radial (principal) quantum number for the baryon isobars.

Finally, in Sec. VIII, we discuss, in general terms, some of the properties of the present model. In connection with the present paper and Ref. 1 and 2, it should be noted that for a few baryon resonances, the fact that they correspond to thresholds for two-particle produc-

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] T. F. Kycia and K. F. Riley, Phys. Rev. Letters 10, 266 (1963).
[2] R. M. Sternheimer, Phys. Rev. Letters 10, 309 (1963).

tion has been known for some time. A general discussion of this situation (as of 1961) has been given in two papers by Salam and by Tuan.[3]

II. THE HYPERON ISOBARS Y_0^* AND Y_1^*

In this section, we will present a scheme similar to that of Ref. 1, which enables one to classify the Y_0^* and Y_1^* isobars of strangeness $S=-1$ by means of appropriate mass relations. In some preliminary work, Kycia and Riley[4] had suggested that the Λ and the states[5] Y_0^* (1405 MeV), Y_0^* (1520 MeV), and Y_0^* (1815 MeV) may form a quadruplet of the same type as those discussed by them in Ref. 1, provided that one uses the ABC particle (with $I=0$, $m\approx 290$ MeV)[6] for two of the links, and a three-pion combination (with $I=0$, $m\approx 410$ MeV) for the other two links. However, difficulties were encountered in trying to fit the two $I=1$ isobars [Y_1^* (1385) and Y_1^* (1660)] into a second system whose ground state would be the Σ hyperon. In the present work, we will use the ABC and 3-π links proposed by Kycia and Riley.[4]

The two other observed $S=-1$ states, namely, Y_1^* (1385) and Y_1^* (1660), are arranged as shown in Fig. 1, by employing a $\pi^{\pm}\pi^0$ link [1] between Λ and Y_1^* (1385), and one-pion links between Y_1^* (1385) and Y_0^* (1520), and between Y_0^* (1520) and Y_1^* (1660). It is easily seen that isotopic spin is conserved for each combination implied by Fig. 1. In the present model, the Σ has no direct connection with the known $S=-1$ isobars, but is connected only via the K and $\bar{K}^*$ links to the

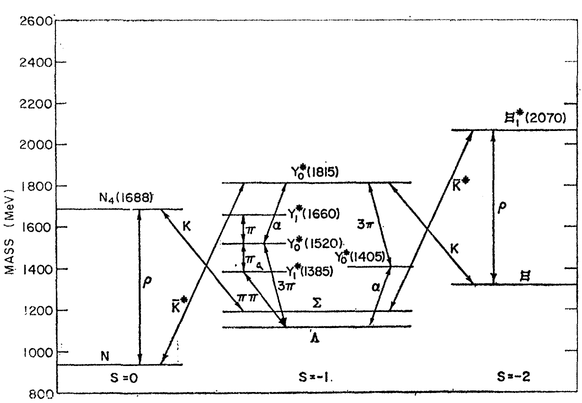

FIG. 1. Energy levels of the baryons and baryon isobars. Only the states with $S=-1$, and those $S=0$ and $S=-2$ states which are involved in the K and $\bar{K}^*$ links are shown. The ABC particle is denoted by α in the figure.

$S=0$ and $S=-2$ states. The accuracy with which the various isobar masses are known is probably ± 5 MeV for Y_1^* (1385), Y_0^* (1405), and Y_0^* (1520); ± 10 MeV for Y_1^* (1660), and ± 20 MeV for Y_0^* (1815). The mass relations proposed in Fig. 1 are satisfied within these experimental uncertainties. One obtains the following expression for the mass m_Λ:

$$m_\Lambda = m_N + m_{K^*} - m_{\mathrm{ABC}} - m_{3\pi}. \quad (1)$$

It may be noted that the K and K^* links of Ref. 2 involve the addition (to the baryon state of lower mass) of either a K meson ($\Delta S=+1$) or a $\bar{K}^*$ particle ($\Delta S=-1$), but not of $\bar{K}$ or K^*. For both K links, the addition of the K meson leads to $\Delta I=-\frac{1}{2}$ for the baryon state. For the $\bar{K}^*$ link from the nucleon to Y_0^* (1815), we also have $\Delta I=-\frac{1}{2}$. If one assumes $\Delta I=-\frac{1}{2}$ for both $\bar{K}^*$ links, then one would predict $I=\frac{1}{2}$ for the hypothetical $S=-2$ state[2] Ξ_1^* (2070) (see Fig. 1).

III. VALIDITY OF THE MASS RELATIONS

In connection with the present results, we wish to point out some consequences of isotopic spin conservation. We consider the N_4 (1688) isobar, which, according to the present model [1,2] can be regarded as either the (N,ρ) or the (Σ,K) combination. The wave function for the $I_z=+\frac{1}{2}$ state can be written as follows:

$$\psi(N_4,I_z=+\tfrac{1}{2}) = -(\tfrac{1}{3})^{1/2}\psi(\Sigma^0,K^+)+(\tfrac{2}{3})^{1/2}\psi(\Sigma^+,K^0). \quad (2)$$

The wave function for $I_z=-\frac{1}{2}$ is given by

$$\psi(N_4,I_z=-\tfrac{1}{2}) = -(\tfrac{2}{3})^{1/2}\psi(\Sigma^-,K^+)+(\tfrac{1}{3})^{1/2}\psi(\Sigma^0,K^0). \quad (3)$$

In view of the previously used mass sum relations, we are led to calculate the masses of the (Σ^0,K^+) and (Σ^+,K^0) combinations which enter into Eq. (2). One finds:[7]

$$m(\Sigma^0,K^+) = (1193.0\pm0.5)+(493.9\pm0.2) = 1686.9\pm0.5 \text{ MeV}, \quad (4)$$

$$m(\Sigma^+,K^0) = (1189.4\pm0.2)+(497.8\pm0.6) = 1687.2\pm0.6 \text{ MeV}. \quad (5)$$

It is seen that the two mass values of Eqs. (4) and (5) are in good agreement with each other, well within the errors of the measurements.

For the $I_z=-\frac{1}{2}$ state, the masses of the two combinations are also in good agreement, since

$$m(\Sigma^-,K^+) = (1197.4\pm0.3)+(493.9\pm0.2) = 1691.3\pm0.4 \text{ MeV}, \quad (6)$$

$$m(\Sigma^0,K^0) = (1193.0\pm0.5)+(497.8\pm0.6) = 1690.8\pm0.8 \text{ MeV}. \quad (7)$$

[3] A. Salam, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles*, 1961 (Centre d'Etudes Nucleaires de Saclay, Seine et Oise, 1961), Vol. 2, p. 191. S. F. Tuan, Nuovo Cimento **23**, 448 (1962).

[4] T. F. Kycia and K. F. Riley (private communication).

[5] The Y_0^* and Y_1^* mass values used in Fig. 1 are those given by S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

[6] N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters **7**, 35 (1961).

[7] W. H. Barkas and A. H. Rosenfeld, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 877; and University of California Radiation Laboratory Report UCRL-8030 (Revised), 1963 (unpublished). W. H. Barkas, J. N. Dyer, and H. H. Heckman, Phys. Rev. Letters **11**, 26 (1963).

It thus appears that the mass sum relation holds not only for the strong interactions, but also in connection with the electromagnetic splittings of the Σ and K particles which are involved in the (Σ,K) combinations for the N_4 isobars. We may expect that the masses of the $I_z=+\frac{1}{2}$ and $-\frac{1}{2}$ states of N_4 are approximately given by the averages of Eqs. (4), (5), and of (6), (7), respectively. One thus obtains $m(N_4,\ I_z=+\frac{1}{2})\cong 1687.0$ MeV, and $m(N_4 I_z=-\frac{1}{2})\cong 1691.0$ MeV. Thus, the mass of N_4 would be higher for $I_z=-\frac{1}{2}$ than for $I_z=+\frac{1}{2}$, in agreement with the similar result for the n-p mass difference.[8]

On the assumption that Y_0^* (1815) can be regarded as the combination (Ξ,K), we obtain the following expression for the wave function:

$$\psi[Y_0^*(1815)]=(\tfrac{1}{2})^{1/2}\psi(\Xi^-,K^+)-(\tfrac{1}{2})^{1/2}\psi(\Xi^0,K^0). \qquad (8)$$

With $m_{\Xi^-}=1321.0\pm0.5$ MeV,[9] one finds

$$m(\Xi^-,K^+)=(1321.0\pm0.5)+(493.9\pm0.2)=1814.9\pm0.5\ \text{MeV}. \qquad (9)$$

According to a recent experiment,[10] there is an indication that the mass of the Ξ^0 is lower by a few MeV than m_{Ξ^-}. On this assumption, the $\Xi^- - \Xi^0$ mass splitting would have the correct sign to compensate approximately for the $K^+ - K^0$ mass difference; i.e., we may have $m(\Xi^0,K^0)\approx m(\Xi^-,K^+)$.

If one uses a value of 1815 MeV for $m_{Y_0^*}$, one deduces from Eq. (2) of Ref. 2 that the mass of K^* is $m_{Y_0^*}-m_N=876$ MeV. This result is probably not outside the limits of error of the present measurements of m_{K^*}, especially in view of the relatively large width of this state ($\Gamma\sim 50$ MeV).

IV. MASS RELATIONS FOR MESONIC PARTICLES

The mass sum relation has been used extensively[1,2] for the baryon isobars. It may be noted that this relation also seems to apply for some of the mesonic particles: (1) Kycia and Riley[1,4] have used $\pi\pi$ and $\eta\pi$ combinations with a mass equal to the sum of the masses of the constituents. With these values of $m_{\pi\pi}$ and $m_{\eta\pi}$, they have obtained good agreement with the masses of the nucleon isobars. (2) The fact that the mass of the η meson is closely equal to $4m_\pi$ has been pointed out previously.[11] One finds that[12] $m_\eta=550\pm3$ MeV is equal to $2m_{\pi^\pm}+2m_{\pi^0}$ within the experimental uncertainty of m_η.

In this connection, we wish to point out two empirical mass relations pertaining to the mesons:

(1) If one uses the value $m_{K^*}=876$ MeV discussed above, and with $m_K=496$ MeV, one finds

$$m_K+m_{K^*}=2m_{\eta\pi}\cong 1372\ \text{MeV}. \qquad (10)$$

(2) Within the experimental uncertainty of the mass of the f_0 meson, one obtains

$$m_{f_0}+m_{\pi^\pm\pi^0}=m_\rho+m_\omega\cong 1532\ \text{MeV}, \qquad (11)$$

where $m_{\pi^\pm\pi^0}=274$ MeV, and $m_\rho=750$ MeV, $m_\omega=782$ MeV are the masses of the ρ and ω mesons, respectively. Equation (11) is satisfied exactly for $m_{f_0}=1258$ MeV. We note that this value of m_{f_0} would fit very closely the mass difference $m_{N_1}-m_N$, for the following reason. As was pointed out by Diddens *et al.*,[13] the mass of the top level (N_6) of the N_1 system,[1] $m_{N_6}=2190\pm20$ MeV, is approximately given by $m_{N_6}=m_N+m_{f_0}$. In view of Fig. 2 of Ref. 1, the N_1-N mass difference is then given by

$$m_{N_1}-m_N=m_{f_0}-m_{\eta\pi}-m_{\pi^\pm\pi^0}. \qquad (12)$$

Upon using $m_{N_1}=1238\pm2$ MeV,[1,5] $m_N=939$ MeV, $m_{\eta\pi}=686$ MeV, $m_{\pi^\pm\pi^0}=274$ MeV, Eq. (12) gives $m_{f_0}=1259\pm2$ MeV.

In connection with the validity of the mass sum rule, it should be pointed out that in the present model, this rule has been verified a total of 19 times, which constitutes strong empirical evidence that the rule is valid and meaningful, and is not the result of accidental coincidences. In addition, we note that the present scheme includes all of the well-known nucleon and hyperon isobars.

The 19 cases of the validity of the mass sum relation are as follows: (1) 7 cases for the nucleon isobars; (2) 5 cases for the hyperon isobars belonging to the Λ system; (3) 3 cases corresponding to the two K-meson links, and the $\bar{K}^*$ link between Y_0^* (1815 MeV) and the nucleon; (4) the η meson; (5) 3 cases, corresponding to the electromagnetic splittings of the Σ, Ξ, and K particles which are involved for the two N_4 states and for Y_0^* (1815 MeV).

V. INTERPRETATION OF THE MASS SUM RELATION

We will now discuss a possible interpretation of the mass sum relation. In analogy to the model of Peierls and Treiman[11] for the η meson, it seems possible to assume that the baryon isobars are very loosely bound nuclei consisting of an unexcited baryon plus one or several mesonic particles. As discussed below, there are 6 isobar systems in the present scheme, whose ground states are: N, Λ, Σ, Ξ, the N_1 isobar (with $I=J=\frac{3}{2}$), and the Ξ^* resonance. The mesonic particles which have been used are as follows: π, $\pi^\pm\pi^0$, 3π, $\eta\pi$, ρ, ABC, f_0, K,

[8] However, it should be pointed out that if one applies the mass relations to $N_4=(N,\rho)$ considering the charge states of the ρ meson, one obtains incorrect results (i.e., a mass difference between ρ^+ and ρ^-).

[9] L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, L. Gray, J. Leitner, S. Lichtman, and J. Westgard, Phys. Rev. Letters **9**, 229 (1962).

[10] J. Leitner and N. P. Samios (private communication).

[11] R. E. Peierls and S. B. Treiman, Phys. Rev. Letters **8**, 339 (1962).

[12] H. Foelsche, E. C. Fowler, H. L. Kraybill, J. R. Sanford, and D. Stonehill, Phys. Rev. Letters **9**, 223 (1962).

[13] A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters **10**, 262 (1963).

and $\bar{K}^*$. The following six mass values are involved: m_π, m_ρ, m_{ABC}, m_{f_0}, m_K, and m_{K^*}.

A possible objection to this interpretation of the mass sum relation for the baryon isobars is that in the cases where a ρ meson, N_1, ABC, or $\bar{K}^*$ particle, or a lower mass isobar is involved, one is using a particle which has a considerable width when observed in the free state, i.e., outside the combinations assumed in the present model. As an example, for the N_4 isobar which has been described as a (N,ρ) combination, one may ask whether the well-known rapid ($\approx 10^{-23}$ sec) decay of the ρ into two pions does not prevent any meaningful interpretation of the N_4 as a quasinucleus consisting of a nucleon plus a ρ meson, since the decay of the ρ meson could occur so rapidly as to prevent the N_4 from having a width Γ and decay modes of its own (other than $N_4 \to N+2\pi$). A related difficulty of the present model is that if one considers the Y_0^* (1520 MeV) isobar as a combination of Y_1^* (1385 MeV) and a pion, one would expect that the width of Y_0^* (1520) would be at least as large as that of Y_1^* (1385), whereas it is in fact considerably smaller ($\Gamma[Y_0^*\ (1520)] \cong 15$ MeV, as compared to $\Gamma[Y_1^*\ (1385)] \cong 50$ MeV). This result for the widths has been previously commented upon, in connection with the observation [14] that the mass of Y_0^* (1520 MeV) corresponds to the threshold for the production of Y_1^* (1385 MeV)$+\pi$.

A possible way of at least partially answering these objections is as follows. We assume that when an unstable particle (with respect to the strong interactions) is in combination to form an isobar, it has a unique and well-defined mass. This assumption can be justified if one regards the isobar as a sort of compound nucleus. In analogy to the usual concept of the compound nucleus in nuclear reactions, it is assumed that the interaction between the constituent particles of the isobar is sufficiently strong, so that the individual constituents lose their identity to some extent, and as a result, the normal decay modes of the unstable particles do not take place while they are in a state of combination to form an isobar. Instead the isobar has a width of its own, which is determined only by the observed decay modes of the isobar, and which is, therefore, not related to the widths of the constituent particles when observed in the free state. In this connection, we note that the decay of an isobar into its constituent particles, e.g., $N_4 \to N+\rho$, or $N_4 \to \Sigma+K$, essentially cannot take place, since the phase space for the decay products would be zero, on account of the mass relation.

If one adopts the present analogy of a quasinucleus for the baryon isobars and some of the mesonic particles, the question still remains as to why the binding energy is so small. In fact, the binding energy must be essentially zero if the mass sum relation is to be rigorously satisfied. In this connection, it may be noted that even for the case of ordinary nuclei, this problem does not seem to have been completely solved. If the binding energy of the deuteron (2.2 MeV) is any indication, then one may expect only very small deviations from the mass relations. It should be noted that deviations of this order of magnitude ($\lesssim$2–3 MeV) would be very difficult to establish experimentally, on account of the much larger widths of the various isobaric states.

[14] S. F. Tuan, Nuovo Cimento **23**, 448 (1962); M. Nauenberg and A. Pais, Phys. Rev. **126**, 360 (1962); W. Krolikowski, Nuovo Cimento **22**, 872 (1961).

VI. CLASSIFICATION OF ISOBAR SYSTEMS

As mentioned above, in the present scheme, there are 6 isobar systems, two for each value of the strangeness S. The ground states of these systems (after which they have been called) are N and N_1 for $S=0$, Λ and Σ for $S=-1$, Ξ and Ξ^* for $S=-2$. In connection with the links between different systems (provided by K, $\bar{K}^*$, and f_0), we can distinguish two types of systems, which will be called type A and type B. The nucleon system (consisting of the nucleon and the isobars N_3, N_4, and N_7 in the notation of Kycia and Riley) is defined to be of type A. In general, those systems for which the (direct or indirect) link to the nucleon system involves the ground state of the system are defined as type A, whereas those systems for which the link to the nucleon involves the top level are defined as type B. The Σ and Ξ systems (together with the nucleon system) belong to type A, while the N_1, Λ, and Ξ^* systems belong to type B.

The preceding classification of the various systems can be justified as follows:

(1) As concerns the Σ and Ξ systems, at present only the Σ and Ξ ground states have been observed. However, there may exist additional states belonging to these systems. It is obvious from Fig. 1 that both systems belong to type A, since the links to the nucleon involve directly the Σ and Ξ ground states.

(2) The N_1 system is linked to the N system only via the top level N_6 $(m_{N_6}=m_N+m_{f_0})$[13], so that the N_1 system belongs to type B.

(3) Similarly, the only connection of the Λ system with the nucleon appears to be via the top level, Y_0^* (1815). Thus the Λ system belongs to type B.

(4) Finally, we wish to discuss the reasons for the probable existence of the Ξ^* system. It is obvious that the Ξ^* cannot be a state belonging to the Ξ system, since the mass difference $m_{\Xi^*}-m_\Xi=1532-1320=212$ MeV is definitely larger than m_π and less than $2m_\pi$. In addition, in analogy to the existence of the two systems both for $S=0$ and $S=-1$, it may be expected that there exists a second system with $S=-2$. In similarity to the N_1 system, the Ξ^* system may be linked to the Ξ particle via the f_0 meson, in such a manner that the top level of the Ξ^* system has a mass $m_\Xi+m_{f_0}$. However, it is also possible that another meson, such as the ω, provides the link between the Ξ and Ξ^* systems. Corresponding to these possibilities, we have considered two possible schemes for the Ξ^* system. Both of these

schemes (which are of type B) would give $m_{\Xi^*}=1540$ MeV, which is quite close to the present experimental value of $m_{\Xi^*}=1532\pm3$ MeV, although somewhat higher.

The question as to the theory which underlies the present scheme involves the problem as to which mesonic particles occur in each particular system. It might perhaps be expected that some mesonic particles occur only in type A systems, while others occur only in type B systems, and still others can belong to systems of both types. The $\eta\pi$ is an example of the last category, since it occurs both in the N and N_1 systems. The ρ meson occurs only in a system of type A (in the N system, but not in the N_1 and Λ systems). On the other hand, the $\pi^{\pm}\pi^0$ combination occurs only in type B systems (in the N_1 and Λ systems, but not in the N system). With added experimental information on the $S=-1$ and $S=-2$ isobars, it may be possible to obtain additional correlations of the mesonic particles with the types of systems (A or B).

A significant result of the present scheme is that while the observed isobars are associated with the thresholds for production of a baryon or baryon isobar plus a mesonic particle, there are many inelastic thresholds which do not lead to the existence of an isobar in the πN or $\bar{K}N$ scattering or interaction. This situation has been previously discussed by Ball and Frazer,[15] who have shown that unless the inelastic production cross section rises sufficiently fast near threshold, there will not be any maximum in the elastic scattering cross section which would correspond to isobar formation. We have also previously[2] commented on the absence of isobars which would correspond to production of $\Lambda+K$, $\Lambda+K^*$, and $N_1+\bar{K}$, even though these thresholds lie in the well-investigated regions of πN and $\bar{K}N$ scattering (see footnote 5 of Ref. 2). Thus, $m_\Lambda+m_K=1611$ MeV, $m_\Lambda+m_{K^*}=2000$ MeV, but there are no $I=\frac{1}{2}$ isobars in the πN scattering at these mass values. Similarly, we have: $m_{N_1}+m_K=1734$ MeV. If one would assume the validity of the $\Delta I=-\frac{1}{2}$ selection rule for the addition of a $\bar{K}$ meson, then the isotopic spin of the $(N_1,\bar{K})$ combination would be $I=1$. An isobar with $I=1$ should be observable in K^-p or K^-n scattering or the corresponding K^-p and K^-n inelastic interactions, but no such isobar has been found in the vicinity of the 1734-MeV mass value. Of course, if one assumes that $\Delta I=-\frac{1}{2}$ does not hold generally for the K-meson links, then the $(N_1,\bar{K})$ combination could have $I=2$, and would be observable only in inelastic interactions of the type $K^-+p\rightarrow(N_1,\bar{K})+\pi$.

Aside from the preceding three examples involving K and K^*, there are numerous examples involving the mesonic particles with $S=0$, which do not involve the formation of an isobar. Among the examples which lie in the well-investigated regions of πN and $\bar{K}N$ scattering, we may mention the following:

$$N+\pi,\ N+2\pi,\ N+3\pi,\ N+\eta,\ N+\omega;$$
$$N_1+\pi,\ N_1+3\pi,\ N_1+\eta,\ N_1+\rho,\ N_1+\omega;$$
$$N_2+\pi,\ N_2+2\pi,\ N_2+\rho;\ N_4+\pi,\ N_4+2\pi,\ N_4+\rho;$$
$$\Lambda+\pi,\ \Lambda+\eta,\ \Lambda+\rho;\ \Sigma+\pi,\ \Sigma+2\pi,\ \Sigma+3\pi,\ \Sigma+\eta.$$

It thus appears that the number of thresholds which do not give rise to isobars vastly exceeds the number of thresholds which correspond to isobar formation. This situation may indicate the existence of a selection rule which determines the possibility of isobar formation. At present, we have not been able to derive a general selection rule of this type.

VII. QUANTUM NUMBERS OF THE ISOBARS

In this section, we will discuss the set of quantum numbers which is necessary to characterize a baryon isobar. For the nucleon isobars, Kycia and Riley[1] have shown that the isotopic spin I and the total angular momentum J are sufficient to identify the isobaric state, on account of a selection rule, namely: $J-L=I-1$, which determines the angular momentum L (or equivalently, the parity of the state). It is implied by their discussion, which is based on the empirical evidence from the πN scattering, that there exists, at most, one isobar for each set of J,L, and I. If this result holds in general (i.e., also for $S=-1$ and $S=-2$), then it would follow that there is no analog to a radial quantum number for the baryon isobars, in contrast to the case of atomic systems. This result would lend support to the view that the distance r between the constituents of an isobar may not have any physical meaning.[16]

Since we must also specify the strangeness S, and since the selection rule of Kycia and Riley[1] seems to be peculiar to the $S=0$ states, we conclude that we must, in general, specify the four quantum numbers I, J, L, and S, in order to characterize an isobaric state.[17]

For the $S=-1$ states, no selection rule of the type used by Kycia and Riley will hold, as can be easily seen from the spin and parity assignments of the Λ, Σ, and Y_1^* (1385). It has been recently demonstrated[18] that the relative $\Sigma-\Lambda$ parity is even. Thus we can define the Λ and Σ hyperons as being both $P_{1/2}$. On the other hand, the Y_1^* (1385) state has been shown[19] to be

[15] J. S. Ball and W. R. Frazer, Phys. Rev. Letters **7**, 204 (1961).

[16] However, it has been pointed out by Dr. W. J. Willis that the energy associated with a possible radial mode of excitation might be very large. In this case, the corresponding isobaric states would have mass values considerably above 2 BeV, which might account for the fact that they have not been observed experimentally.

[17] It is, of course, understood that the z component of isotopic spin, I_z, will determine the charge state of the isobar, so that, strictly speaking, five quantum numbers (I, I_z, J, L, S) are necessary to determine a given isobar charge state.

[18] H. Courant, H. Filthuth, P. Franzini, R. G. Glasser, A. Minguzzi-Ranzi, A. Segar, W. Willis, R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters **10**, 409 (1963).

[19] J. B. Shafer, J. J. Murray, and D. O. Huwe, Phys. Rev. Letters **10**, 179 (1963).

$P_{3/2}$. A possible generalization of the selection rule of Ref. 1 would be: $J-L=I-\frac{1}{2}$ (for $S=-1$). This rule would work for the Λ and Y_1^* (1385), but not for the Σ. In fact, the use of the same assignment ($P_{1/2}$) for Λ and Σ indicates that for the $S=-1$ states, both I states with the same values of L and J are possible. In this connection, we note that the assignment[20] for Y_0^* (1520) is $D_{3/2}$, while Y_1^* (1660) is believed to be either $P_{3/2}$ or $D_{3/2}$.[21] Since we already have a $P_{3/2}$ state with $I=1$ (the 1385-MeV isobar), it would be necessary to have $D_{3/2}$ for Y_1^* (1660),[22] if there is to be only one state with given values of I, J, and L. No experimental evidence seems to be available for Y_0^* (1405 MeV). According to the present scheme, it could be $S_{1/2}$ or $P_{3/2}$. Finally, the Y_0^* (1815 MeV) isobar is believed to have $J \geqq \frac{5}{2}$, and could be either $D_{5/2}$ or $F_{5/2}$.

VIII. DISCUSSION

If one is looking for a dynamical model of the baryon isobars, it seems appropriate to consider again the mass sum relation. As discussed above, from the work of Ref. 1 and 2 and the present paper, one has obtained expressions for the masses of 16 baryon states in terms of the nucleon mass and the masses of only 6 mesons, namely: π, ρ, ABC, f_0, K, and $\bar{K}^*$. It has been shown that a possible interpretation of the mass sum relation can be given, according to which the baryon and the mesonic particle enter into combination to form a sort of nucleus. The fact that the mass sum relation holds also for some mesonic particles, in addition to the baryon isobars, may indicate that there exists an underlying tendency for the strongly interacting particles, regardless of whether they are baryons or mesons, to form loosely bound nuclei.

The existence of the K, $\bar{K}^*$, and f_0 links implies that the states which are involved in the links can be represented in terms of several alternate combinations.[23] Thus, we have

$$N_4=(N,\rho)=(\Sigma,K), \tag{13}$$

$$Y_0^*\ (1815)=(N,\bar{K}^*)=(\Xi,K)=(\Lambda,\ 3\pi,\ \mathrm{ABC}), \tag{14}$$

$$N_6=(N_1,\ \eta\pi,\ \pi^\pm\pi^0)=(N,f_0). \tag{15}$$

We note that Y_0^* (1815) can actually be represented in three different ways, all of which involve different constituents. In addition, the last combination, (Λ, 3π, ABC), can be written in two alternate ways, as discussed in Ref. 23.

When more than one combination is possible, as in (13)–(15), the isobar can perhaps resonate several times between the alternate configurations, before its ultimate decay takes place. This situation is also somewhat similar to that which exists for the case of ordinary nuclei, which for some purposes can be regarded as made up of α particles, or of deuterons, or of nucleons arranged in shells, which move in an average effective potential (shell model or independent-particle model). It is quite conceivable that the nucleus resonates at a rapid rate between all of these possible structures.

Finally, it should be pointed out that the present model fulfills to some extent the expectation that all strongly interacting particles (e.g., baryon ground states and excited states) are, in a certain sense, equally fundamental and can be treated on an equal footing. As an example, we note that both the K meson, which is stable with respect to the strong interactions, and the $\bar{K}^*$ particle, which is unstable, can participate in isobar formation. Similarly, in the nucleon system, we have $N_4=(N,\rho)$ and $N_7=(N_3,\rho)$, which shows that both the nucleon and the isobar N_3 can enter into combination in the same way with the ρ meson to form a higher mass isobar. Moreover, the fact that among the 6 baryon ground states, there are two isobars (N_1 and Ξ^*) indicates that also in this respect the isobars and the stable particles (as concerns the strong interactions) can perform a similar role.

It should be emphasized that except for the discussion of Secs. V and VIII concerning a possible interpretation of the mass relations, the results presented above are largely empirical, being based on the observed properties of the baryon resonances.

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[20] M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters, **8**, 28 (1962); and Phys. Rev. (to be published).

[21] P. L. Bastien and J. P. Berge, Phys. Rev. Letters **10**, 188 (1963).

[22] The assignment $D_{3/2}$ for Y_1^* (1660 MeV) is also that given by S. F. Tuan [Phys. Rev. **125**, 1761 (1962)] in his prediction of the possible existence of this isobaric state.

[23] This occurs also for the top levels of the N, N_1, and Λ systems, if one considers the two different intermediate states within the same system, i.e., $N_7=(N_4,\eta\pi)=(N_3,\rho)$; $N_6=(N_2,\eta\pi)=(N_5,\pi^\pm\pi^0)$; $Y_0^*(1815)=(Y_0^*(1405),3\pi)=(Y_0^*(1520),\mathrm{ABC})$.